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Nonlinear dynamics of electro-mechanical system composed of two pendulums and rotating hub

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Abstract

Dynamics of a structure composed of two pendulums attached to a rotating rigid hub is analysed in the paper. The system is rotating in a horizontal plane so the gravity force does not influence its motion. Both pendulums are treated as lumped masses fixed at the ends of stiff and massless rods connected to the hub by flapping hinges. Three possible variants of the excitation are considered: (a) an ideal energy source, where torque is given by a defined harmonic function (b) a simplified limited power supply, where torque is given by an arbitrary defined function or (c) a non-ideal energy source, where DC output characteristic of limited power is taken into account.

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Keywords: Rotating structure; non-ideal energy source; pendulum; nonlinear dynamics

1. Introduction

Rotating structures are very well known in mechanical and aerospace engineering. Typical examples of that kind of structures are wind turbines, helicopter rotors and jet engines. In the paper [1] an investigation of a nonlinear model of a flexible slewing beam oscillating in longitudinal and flexural direction has been presented. A model of the rotating beams with application to helicopters has been presented in paper [2]. In that case blades have been treated as flexible bodies with possibility to flexural and torsional vibrations.

A non-ideal energy source supplied to the system by a DC motor has been consider in paper [3]. The Sommerfeld effect resulting from a limited power supply has been examined by the numerical and analytical analysis.

The synchronization of motion of the rotating pendulums has been intensively investigate in paper [4]. Moreover, there has been experimentally examine the double pendulum system in a synchronous rotation with a possible transition to chaotic motion.

The goal of the present paper is to study the system composed of two pendulums attached to the hub rotating in a horizontal plane. Therefore, the gravity force does not influence the motion of structure, so the problem is different from the dynamics of the classical pendulum system presented in e.g. [5]. The offered model might be seen as a

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simplification of a structure composed of a rotating hub with attached blades [6]. Within the frame of the presented studies the regions of possible synchronized and chaotic motion are examined for various models of a energy source.

2. Model of rotating system

Model of a rotor composed of a flexible beam and a flapping hinge attached to a hub can be reduced to a simple lumped mass model. Blade motion can be simplified to the motion of a solid body, as a pendulum, with a mass and a length corresponding to a selected mode. The Duffing spring and a viscous damper in a joint correspond to a real flapping hinge.

Model of the structure rotating in a horizontal plane consists of two pendulums attached to a rigid hub. Each of the pendulums is composed of a lumped mass fixed to a stiff and massless rod connected to the hub by a joint treated as a flapping hinge represented by a nonlinear Duffing type spring and a viscous damper (Fig. 1a, b). The radius and the mass moment of inertia of the hub are denoted by R_0 and J_0 respectively. The parameters m_j, l_j are mass and length of the pendulum, c_j is a viscous damping coefficient and k_j, k_j^* are the coefficients of a nonlinear Duffing type spring, where in our case $j = 1, 2$ (see Fig. 1b). The hub may rotate or oscillate in the horizontal plane and its current position is described by an angle ψ.

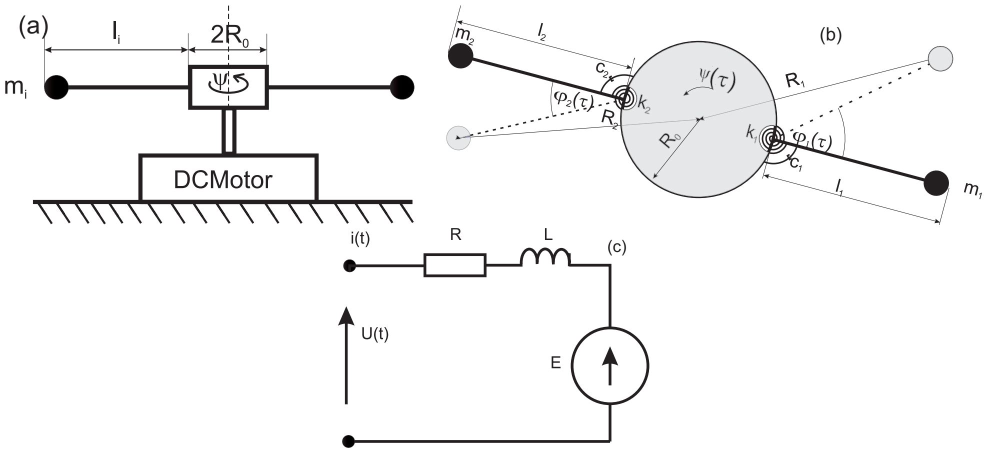

Fig. 1. Schematic diagram of the model with two pendulums attached to a rotating hub (a) front view; (b) view from the top; (c) electrical circuit of DC motor.

The torque driving the system under consideration is generated by the DC motor, which we consider in three variants: (a) as an ideal energy source where the output torque is defined by a time dependent function, (b) a simplified limited power supply, where torque is given by a time and angular velocity dependent function or (c) as limited power supply source, where the actual motor characteristic (Fig. 1c) is taken into account. Each pendulum may oscillate around the hinge and its relative motion, which is described by the coordinate φ_j defined with respect to floating frame of reference rotating with the hub. We note that constraints in both hinges allow not only oscillations but also a full rotation of the pendulums in their relative motion.

The DC motor is described by parameters R, L, c_ϕ - resistance, inductance and electric constant depending on stator magnetic flux, respectively. The presented model is strongly nonlinear and different from the classical pendulums systems, because the gravity force does not influence the horizontal plane motion.

3. Eqation of motion

The differential equations of motion are derived from Lagrange equation of the second kind. Energy, kinetic T, potential V and Rayleigh dissipation function D for a general case of N_p pendulums system are defined as

$$\begin{aligned} T &= \frac{1}{2}J_0\left(\frac{d\psi}{dt}\right)^2 + \sum_{j=1}^{N_p}\left[\frac{1}{2}m_jR_j^2\left(\frac{d\psi}{dt}\right)^2 + \frac{1}{2}m_jl_j^2\left(\frac{d\varphi_j}{dt}\right)^2 + l_jR_j\cos\beta_j\frac{d\varphi}{dt}\frac{d\psi}{dt}\right] \\ V &= \sum_{j=1}^{N_p}\left(\frac{1}{2}k_j\varphi_j^2 + \frac{1}{4}k_j^*\varphi_j^4\right) \qquad D = \sum_{j=1}^{N_p}\left[\frac{1}{2}c_j\left(\frac{d\varphi_j}{dt}\right)^2\right] \end{aligned} \tag{1}$$

The governing equations of motion are obtained from the Lagrange equation

$$\begin{aligned} \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\varphi}_j}\right) - \frac{\partial T}{\partial \varphi_j} + \frac{\partial V}{\partial \varphi} + \frac{\partial D}{\partial \dot{\varphi}_j} &= 0 \qquad\qquad j = 1, \ldots, N_p \\ \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\psi}}\right) - \frac{\partial T}{\partial \psi} + \frac{\partial V}{\partial \psi} + \frac{\partial D}{\partial \dot{\psi}} &= M\left(t, \psi, \dot{\psi}\right) \end{aligned} \tag{2}$$

Next, all equations are transformed to the dimensionless form by introducing dimensionless time $\tau = \omega_{01}^* t$, where t is time and $\omega_{01}^* = \sqrt{k_1/m_1 l_1^2}$ is a natural, linear frequency of the first pendulum. Considering both pendulums, the rotating hub and characteristic of DC motor four dimensionless differential equations of motion have the form:

$$\begin{aligned} &(1+\gamma_1+\gamma_2)\ddot{\psi} + \zeta_h\dot{\psi} + 2\frac{\gamma_1}{\delta_1}\dot{\delta}_1\dot{\psi} + 2\frac{\gamma_2}{\delta_2}\dot{\delta}_2\dot{\psi} + \frac{\gamma_1}{\delta_1}\cos\beta_1\ddot{\varphi}_1 + \frac{\gamma_2}{\delta_2}\cos\beta_2\ddot{\varphi}_2 \\ &+\frac{\gamma_1}{\delta_1^2}\cos\beta_1\dot{\delta}_1\dot{\varphi}_1 + \frac{\gamma_2}{\delta_2^2}\cos\beta_2\dot{\delta}_2\dot{\varphi}_2 + \frac{\gamma_1}{\delta_1}\frac{d}{d\tau}(\cos\beta_1)\dot{\varphi}_1 + \frac{\gamma_2}{\delta_2}\frac{d}{d\tau}(\cos\beta_2)\dot{\varphi}_2 = \alpha i^* \\ &\ddot{\varphi}_1 + \delta_1\cos\beta_1\ddot{\psi} + \delta_1\frac{d}{d\tau}(\cos\beta_1)\dot{\psi} \\ &+\cos\beta_1\dot{\delta}_1\dot{\psi} - \delta_1\frac{d\delta_1}{d\varphi_1}\dot{\psi}^2 - \cos\beta_1\dot{\delta}_1\dot{\psi} - \delta_1\frac{d}{d\tau}(\cos\beta_1)\dot{\psi} + \zeta_1\dot{\varphi}_1 + \omega_{01}^2\varphi_1 + \kappa_1\omega_{01}^2\varphi_1^3 = 0 \\ &\ddot{\varphi}_2 + \delta_2\cos\beta_2\ddot{\psi} + \delta_2\frac{d}{d\tau}(\cos\beta_2)\dot{\psi} + \cos\beta_2\dot{\delta}_2\dot{\psi} - \delta_2\frac{d\delta_2}{d\varphi_2}\dot{\psi}^2 \\ &-\cos\beta_2\dot{\delta}_2\dot{\psi} - \delta_2\frac{d}{d\tau}(\cos\beta_2)\dot{\psi} + \zeta_2\dot{\varphi}_2 + \omega_{02}^2\varphi_2 + \kappa_2\omega_{02}^2\varphi_2^3 = 0 \\ &-\alpha_{m1}i^* - \alpha_{m2}\frac{d\psi}{d\tau} + \alpha_{m3} = \frac{di}{d\tau} \end{aligned} \tag{3}$$

where:

$$\omega_{0j} = \frac{\omega_{0j}^*}{\omega_{01}^*}, \quad \omega_{01} \equiv 1, \quad \delta_{0j} = \frac{R_0}{l_j}, \quad \delta_j = \frac{R_j}{l_j} = \sqrt{\delta_{0j}^2 + 1 + 2\delta_{0j}\cos\varphi_j}, \quad \cos\beta_j = \sqrt{1 - \frac{\delta_{0j}^2}{\delta_j^2}\sin^2\varphi_j},$$

$$\gamma_{0j} = \frac{m_j l_j^2}{J_0}, \quad \gamma_j = \gamma_{0j}\delta_j^2, \quad \kappa_j = \frac{k_j^*}{k_1}, \quad \zeta_j = \frac{c_j}{m_j l_j^2 \omega_{01}^*}, \quad \zeta_h = \frac{c_j}{J_0\omega_{01}^*}, \quad \mu = \frac{M}{J_0\omega_{01}^{*2}}, \quad j = 1, 2,$$

$$\alpha_1 = \frac{R}{L\omega_{01}}, \qquad \alpha_2 = \frac{c_\varphi}{Li_{max}}, \qquad \alpha_3 = \frac{U}{Li_{max}\omega_{01}}, \qquad \alpha = \frac{c_\varphi i_{max}}{k_1}$$

The derived mathematical model is general and allow studying dynamical behavior of rotating structures, nonlinear phenomena and bifurcations of the system.

4. Numerical and analytical studies

The dimensionless differential equations of motion (3) are strongly nonlinear. Therefore, dynamics of the system is studied by direct numerical simulations. Regions of chaotic motion of the system have been found for the system with ideal energy source as well as for the system with simplified limited power supply.

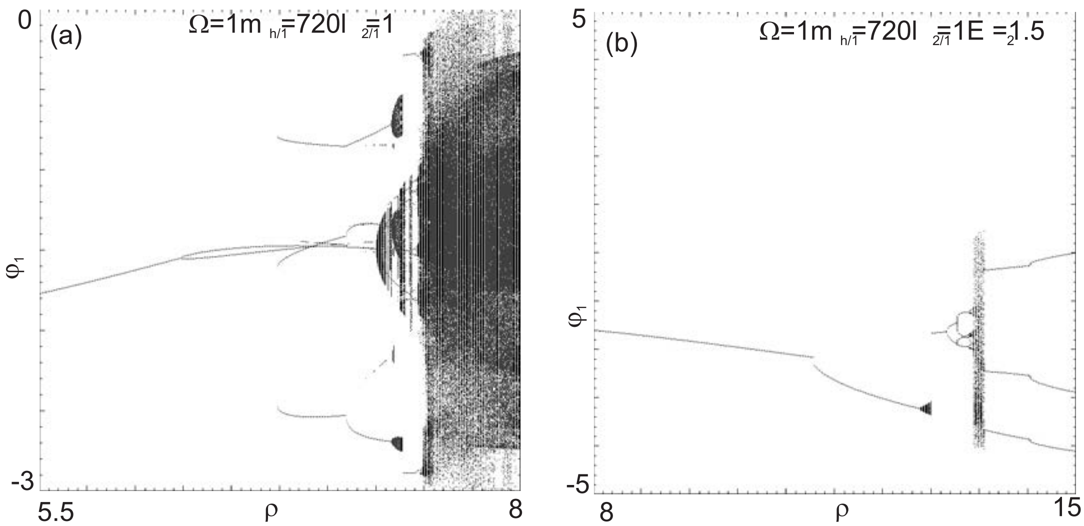

Fig. 2. Bifurcation diagram of φ_1 against excitation amplitude ρ and fixed frequency $\Omega = 1$ (a) zoom of the first chaotic region ideal energy source; (b) simplified limited power supply.

In case of ideal energy source the driven torque is defined by harmonic function $\mu = \rho\cos(\Omega\tau)$. The simplified non-ideal energy source is given by $\mu = E_1 - E_2\dot{\psi}$, where E_1 is an arbitrary harmonic function and the parameter E_2 is connected with output characteristic of a DC motor. At the both diagrams given bellow (Fig. 3a, b) the dark zones represent systems exhibiting chaotic motion. This has been proved by Poincar'e maps.

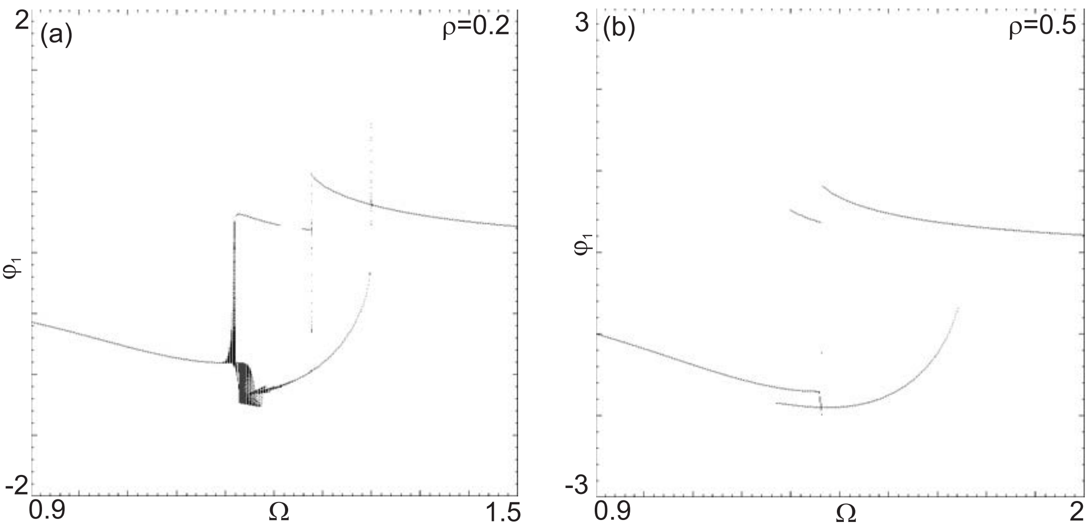

Fig. 3. Resonance curve for selected excitation amplitudes for the system with DC motor (a) $\rho = 0.2$; (b) $\rho = 0.5$.

In case of the non-ideal energy source, as a DC motor, the solutions of equation of motion (3) are found numerically assuming periodic oscillation of the voltage: $U = \rho\cos(\Omega\tau)$ (Eq. 3d). Resonance curves shown at Fig. 3 present more than one solution. In case of amplitude of excitation $\rho = 0.2$ unstable regions have been found. Initial results of the studied system have been presented in the former authors paper [7].

Dynamics of the system is also studied analytically by the harmonic balance method. For the purpose of this analysis we assume that the system is weakly nonlinear and symmetric (no pendulums detuning).

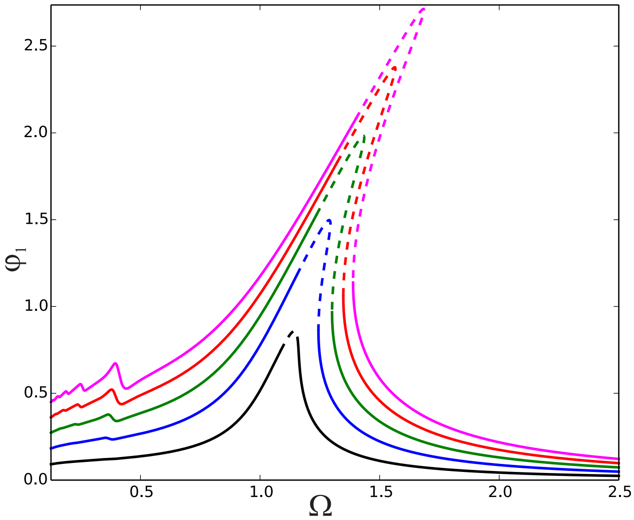

Fig. 4. Resonance curve for selected excitation amplitudes of φ_1 where: $\rho = 0.2$ blue, $\rho = 0.3$ green, $\rho = 0.4$ red, $\rho = 0.5$ violet.

Moreover, the driving torque is treated as the ideal energy source, defind by a harmonic function $\mu = \rho\cos(\Omega\tau)$. Selected resonance curves obtained for different amplitudes of excitation are given in figure 4, where dashed lines represent unstable regions. The applied harmonic balance method showed more solutions than the exhibited by initial numerical studies what is presented at resonance curves in figure 5.

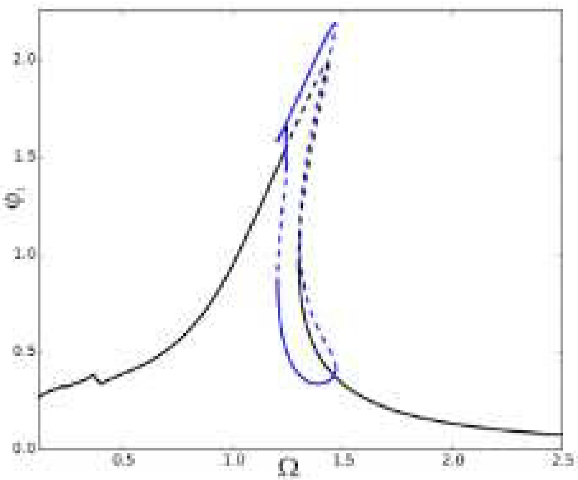

Fig. 5. Resonance curve for selected excitation amplitudes of φ_1 and $\rho = 0.3$.

The analysis of the system performed by the harmonic balance method has shown that close to the top of the resonance curve the solution is unstable. Furthermore, additional stable solutions may exist (Fig. 5). The resonance curves obtained for the excitation amplitude $\rho = 0.3$ exhibit an additional loop near the unstable region (plotted in blue). For $\Omega = 1.4$ seven different solutions, three stable and four unstable ones have been found. The obtained solutions are in agreement with bifurcation diagrams obtained by direct numerical simulations.

5. Conclusions

The derived mathematical model of rotating hub with two pendulums, represented by the strongly nonlinear differential equations of motion has been studied considering various models of the energy source. The analysed bifurcation scenario showed possible harmonic oscillations, then period doubling and finally the zone of chaotic motion in both cases of ideal and simplified non-ideal energy source. In the system with simplified limited power supply the zone of chaotic motion is smaller. Furthermore, when the system is composed of two pendulums attached to the rotating hub the unstable regions arise on the resonance curves. This phenomenon is observed for amplitude of excitation $\rho = 0.2$ or higher. For higher amplitude, $\rho = 0.5$ more than one solution has occurred.
Even for the model with ideal energy source unstable zones on the resonance curve have been observed (Fig. 4) when the amplitude of excitation increased. As demonstrated in Fig. 5 the double pendulum rotor could have a few stable and unstable solutions.
Preformed numerical and analytical calculations have shown a good mutual agreement.

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References

[1] A. Fenili, J.M. Balthazar, R.M.L.R.F. Brasil, On the mathematical modelling of beam-like flexible structure in slewing motion assuming nonlinear curvature, JSV 268 (2003) 825–828.
[2] M.R.M. Crespo da Silva, D.H. Hodges, Nonlinear Flexure and Torsion of Rotating Beams With Application to Helicopter Blade, Vertica 10 (1986) 1225–1234.
[3] P.J.P. Goncalves, M. Silveira, B.R. Pontes Junior, J.M. Balthazar, The dynamic behavior of a cantilever beam coupled to a non-ideal unbalanced motor through numerical and experimental analysis, JSV 333 (2014) 5115–5129.
[4] J. Strzalko, J. Grabski, J. Wojewoda, M. Wiercigroch, T. Kapitaniak, Synchronous rotation of the set of double pendulum: experimental observations, Chaos. 22 (2012)
[5] T. Kapitaniak, J. Kurths, Synchronized pendula: from Huygens Clocks to chimera states, EPJ Special Topics 223 (2014) 609–612.
[6] A. Fenili, J.M. Balthazar, Some remarks on nonlinear vibrations of ideal and nonideal slewing flexible structures, JSV 282 (2005) 543.
[7] J. Warminski, Z. Szmit, J. Latalski, Nonlinear dynamics and synchronization of pendula attached to a rotating hub, EPJ Special Topics. 223(4) (2014) 827–847.